JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY ANANTAPUR
ANANTAPUR

Course Structure and Syllabi for Pre M. Phil
MATHEMATICS (2009-10)

	PAPER	PAPER CODE
PAPER 1	**Topics in Analysis**	**09PH54101**
PAPER 2	**Analytical Number Theory**	**09PH54202**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY ANANTAPUR ANANTAPUR

Pre-M. Phil MATHEMATICS

(09PH54101) TOPICS IN ANALYSIS

UNIT – I

Abstract Integration: Set-theoretic notations and terminology – The concept of measurability – Simple functions – Elementary properties of measures – Arithmetic in [0, ∞] – Integration of positive functions – Integrations of complex functions – The role played by sets of measure zero

UNIT – II

Positive Borel Measures: Vector spaces – Topological preliminaries – the Riesz representation theorem – Regularity properties of Borel measures – Lebesgue measure – Continuity properties of measurable functions.

UNIT – III

L^p-Spaces: Convex functions and inequalities – The L^p-spaces – Approximation by continuous functions.

UNIT - IV

Elementary Hilbert Space Theory: Inner products and linear functionals – Orthonormal sets – Trigonometric series.

UNIT - V

Examples of Banach Space Techniques: Banach spaces – Consequences of Baire's theorem – Fourier series of continuous functions – Fourier coefficients of L^1 – functions – The Hahn-Banach theorem – An abstract approach to the Poisson integral.

UNIT – VI

Approximation by Rational Functions: Preparation – Runge's theorem – The Mittag-Leffler theorem – Simply connected regions

UNIT-VII

Zeros of Holomorphic Functions: .Infinite products – The Weierstrass factorization theorem – An interpolation problem – Jensen's formula – Blaschke products – The Muntz-Szasz theorem

UNIT-VIII

Analytic continuation: Regular points and singular points – Continuation along curves – The monodromy theorem – Construction of a modular function – The Picard theorem

References:

1. **Walter Rudin, Real & Complex Analysis**, Third Edition, Tata McGraw-Hill Edition (Chapters 1, 2, 3, 4, 5, 13, 15 and 16)

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Pre-M. Phil MATHEMATICS

(09PH54202) ANALYTICAL NUMBER THEORY

UNIT-I:

Prime numbers – The fundamental theorem of arithmetic the series of reciprocals of the primes. The Euclidean algorithm. The greatest common divisor of more than two numbers.

Arithmetical Functions and Dirichlet multiplication:The Mobius function – The Euler quotient function – A relation connecting and – A product formula for the Dirichlet product of arithmetical functions – The Mangoldt function – Multiplicative functions – Multiplicative functions and Dirichlet multiplications – The inverse of a completely multiplications – The inverse of a completely multiplicative function – Liouville's function.

UNIT-II:

Definition and basic properties of congruence – Residue classes and complete residue system – Linear congruence – Residue systems and the Euler Fermat theorem – Polynomial congruence module/Lagrange's theorem – applications of Lagrange' theorem – Simultaneous linear congruence – The Chinese remainder theorem – Applications of the Chinese remainder theorem – Polynomial congruence with prime power module.

UNIT-III:

Averages of Arithmetical Functions: The big oh notation, Asymptotoic equality of functions – Euler's summation formula – Some elementary asymptotic formulas – The average order of d(n)- The average order of the divisor functions $\sigma_\alpha(n)$ – An application to the distribution of lattice points visible form the origin – The average order of $\mu(n)$ and of (n) – the partial sums of a Dirichlet product – Applications to $\mu(n)$ and of $\Lambda(n)$ – Another identity for the partial sums of a Dirichlet product.

UNIT- IV

Some Elementary Theorems on the Distribution of Prime Numbers: Introduction – Chebyshev's functions $\psi(x)$ and $\theta(x)$ – Relations connecting $\theta(x)$ and $\pi(x)$ – Some equivalent forms of the prime number theorem – Inequalities for $\pi(n)$ and p_n – Shapiro's Tauberian theorem – Applications of Shapiro's theorem – An asymptotic formula for the partial sume - The partial sums of the Mobius function – Selberg's asymptotic formula.

UNIT- V

Finite Ableian Groups and Their Characters: Definitions – Examples of groups and subgroups – Elementary properties of groups – Construction of subgroups – Characters of finite abelian lgroups – The character group – The orthogonality relations for characters – Dirichlet characters – Sums involving Dirichlet characters – The nonvanishing of $L(1, \chi)$ for real nonprincipal χ- Dirichlet's Theorem on Primes in Arithmetic Progressions – Introduction – Direichlet's theorem for primes of the form 4n – 1 and 4n + 1 – The plan of the proof of Dirichlet's theorem and its proof.

UNIT- VI

Analytic properties of Dirichlet series – Dirichlet series with nonnegative coefficients – An integral formula for the partial sums of a Dirichlet series

UNIT – VII

The Functions $\zeta(S)$ and $L(S, \chi)$ – Introduction – Properties of the gamma function – Integral representation for the Hurwitz zeta function – A countour integral representation for the Hurwitz zeta function – The analytic continuation of the Hurwitz zeta function – Analytic continuation of $\zeta(s)$ and $L(s, \chi)$ – Hurwitz's formula for $\zeta(s, a)$ – the functional equation for the Riemann zeta function – A functional equation for the Hurwitz zeta function – The functional equation for L-functions – Evaluation of $\zeta(-N, a)$.

UNIT – VIII

Analytic Proof of the Prime Number Theorem: The plan of the proof – Lemmas – A contour integral representation for $\psi_1(x)/x^2$ – Upper bounds for $|\zeta(s)|$ and $|\zeta'(s)|$ near the line $\sigma = 1$ – Inequalilties of $|1/\zeta(s)|$ and $|\zeta'(s)/\zeta(s)|$ – Completion of the proof of the prime number theorem

References:

Tom M.Apostol, Introduction to Analytic Number Theory, Springer International Student Edition, USA